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Optimum Design of Laminated Composite Plates Using Lamination Parameters

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Abstract

A NEW and unified optimal design method is established. When laminated composite plates are symmetric and orthotropic, their in-plane and flexural stiffnesses become the functions of the lamination parameters that are the functions of their stacking sequences. We use the lamination parameters as fundamental design variables in designing laminates. The feasible region of the lamination parameters is obtained on a two-dimensional plane. Optimum design points can be obtained from the geometric relations between the feasible region and an objective function. Optimum designs for required in-plane stiffness, maximum bending stiffness, buckling strength, and natural frequency of laminated plates are performed by using the proposed method. The proposed method is found to be useful for the optimum design of laminated sandwich plates and hybrid plates also. The method can be extended for multiple objective problems and strength-related problems.

Contents

The studies on tailoring laminated fibrous composites have been mainly based on a very simple stacking sequence or the use of the mathematical programming method. Therefore, they do not give the general view of the optimal design for laminated plates. The proposed method is capable of giving general and analytical results for many stiffness and strength-related optimization problems.

The stacking sequence of the laminate of which each lamina is orthotropic is represented by

$$[(\pm\theta_n)_{Nn}/\cdots/(\pm\theta_2)_{N2}/(\pm\theta_1)_{N1}]_S \tag{1}$$

The elements of the normalized in-plane stiffness matrix $[A^*]$ $(=[A]/h)$ and the normalized flexural stiffness matrix $[D^*]$ $(=12[D]/h^3)$ are[1]

$$\begin{Bmatrix} A_{11}^* \\ A_{22}^* \\ A_{12}^* \\ A_{66}^* \end{Bmatrix} = \begin{bmatrix} U_1 & V_1^* & V_2^* \\ U_1 & -V_1^* & V_2^* \\ U_4 & 0 & -V_2^* \\ U_5 & 0 & -V_2^* \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_3 \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} D_{11}^* \\ D_{22}^* \\ D_{12}^* \\ D_{66}^* \end{Bmatrix} = \begin{bmatrix} U_1 & W_1^* & W_2^* \\ U_1 & -W_1^* & W_2^* \\ U_4 & 0 & -W_2^* \\ U_5 & 0 & -W_2^* \end{bmatrix} \begin{Bmatrix} 1 \\ U_2 \\ U_3 \end{Bmatrix} \tag{3}$$

where

$$V_1^* = \frac{2}{h}\int_0^{h/2} \cos 2\theta \, dz, \qquad V_2^* = \frac{2}{h}\int_0^{h/2} \cos 4\theta \, dz \tag{4}$$

$$W_1^* = \frac{24}{h^3}\int_0^{h/2} \cos 2\theta z^2 \, dz, \qquad W_2^* = \frac{24}{h^3}\int_0^{h/2} \cos 4\theta z^2 \, dz \tag{5}$$

The various parameters V^* are called in-plane lamination parameters (ILP), and the various W^* are called the flexural lamination parameters (FLP). They are the only factors that connect laminate stacking sequences with the stiffness of plates in case the ply materials are given. The most important aspect to establish an approach where the lamination parameters are the basic design variables is to clarify the allowable region of the parameters.

The feasible regions of ILP and FLP are obtained as follows[2,3]:

$$V_2^* \geq V_1^{*2} - 1, \qquad V_2^* \leq 1 \tag{6}$$

$$W_2^* \geq W_1^{*2} - 1, \qquad W_2^* \leq 1 \tag{7}$$

The feasible region of FLP is represented as region ABC in Fig. 1, which has the same shape as the feasible region of ILP. The lower boundary curve ABC represents the feasible region of FLP of angle-ply laminates $[(\pm\theta)_{Nh}]_S$, and any point in the region is found to be realized by using two kinds of orientation angles.

The key concept of the proposed method is that optimum design is performed by using the geometric relations between the feasible region and an objective function. For example, the buckling strength of a laminated plate is a function of its flexural stiffness and can be maximized using FLP.

The buckling load $(N_x)_{cr}$ for uniaxial loading conditions is represented as[4]

$$(N_x)_{cr} = \frac{\pi^2}{m^2a^2}[D_{11}m^4 + 2(D_{11} + 2D_{66})m^2R^2 + D_{22}R^4] \tag{8}$$

where $R = a/b$, a is the length, b is the width, and m is the half wave number.

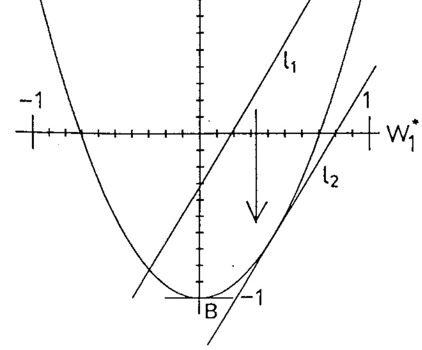

Fig. 1 Feasible region of the flexural lamination parameters and the contour curve of buckling stress.

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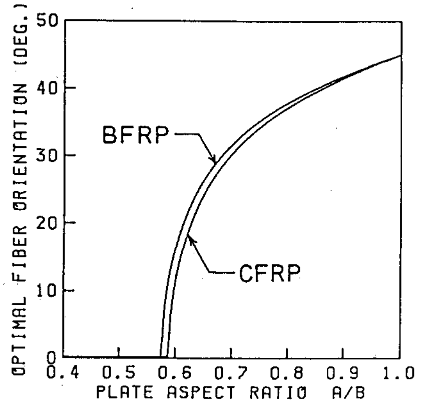


Fig. 2 Optimum orientation angles of boron and carbon fiber reinforced plastics for maximum buckling stress.

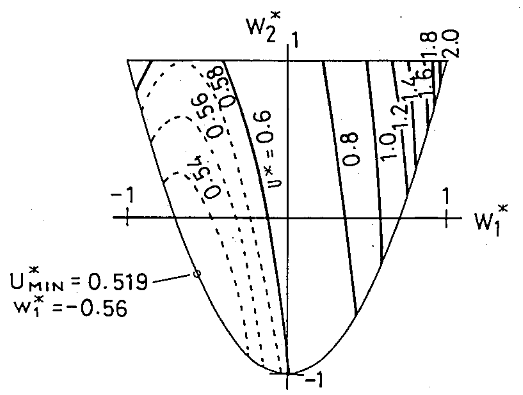


Fig. 3 Contour curves of strain energy for centrally applied concentrated load.

Substituting Eq. (3) into Eq. (8) and setting $m = 1$, that is, for the plate with the aspect ratio R being less than unity, yield a linear relationship between W_1^* and W_2^*, as follows:

$$(N_x)_{cr} = \left[\frac{12b^2(N_x)_{cr}}{\pi^2 h^3}\right][\alpha U_2 W_1^* + (\beta - 6)U_3 W_2^* + \beta U_1 + 2U_4 + 4U_5] \tag{9}$$

where

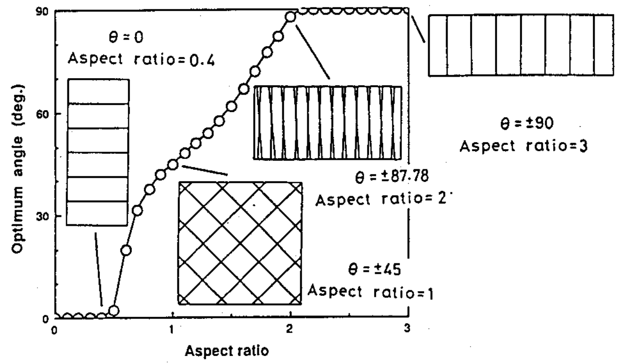
$$\alpha = \frac{1}{R^2} - R^2, \qquad \beta = \frac{1}{R^2} + R^2 \tag{10}$$

The contour curve of the normalized buckling stress becomes a straight line on the FLP plane as shown in Fig. 1. Consequently, the maximization of the buckling stress corresponds to the translation of the line at its maximum in the feasible region of FLP. The optimum design point is obtained as the point of contact between the line and the boundary curve of the feasible region. It should be noted that the optimum laminate construction becomes angle ply since the optimum design point is located on the lower boundary curve.

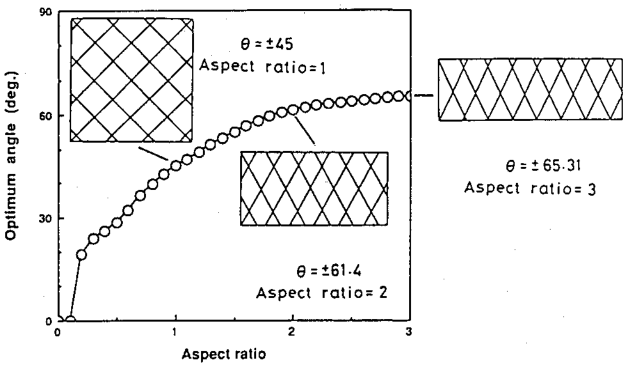
Figure 2 shows the optimum fiber orientations for boron and carbon composites as functions of the plate aspect ratio. When the plate aspect ratio is greater than unity, the determination of the half wave number introduces some complexity into the analysis, but such a problem can also be solved analytically by using FLP.

The maximum stiffness design of laminated plates subject to transversal loading is also performed by using FLP. The maximum stiffness of a structure under any loading condition corresponds to the minimum strain energy stored in the structure. The strain energy of a plate is calculated as

$$U = \frac{1}{2}\int_0^b\int_0^a \{\kappa\}^T[D]\{\kappa\}\ dx\ dy \tag{11}$$



a) Uniformly distributed loading



b) Centrally applied concentrated loading

Fig. 4 Optimum fiber orientation angle as a function of the plate aspect ratio for maximum flexural stiffness.

The curvature of the plate, κ, under any loading condition is obtained by using the Ritz's method. Flexural stiffness D is a function of FLP, and the strain energy can be evaluated on the FLP plane.

Figure 3 shows the contour curves of the strain energy of the laminated plate subject to centrally applied concentrated load. It should be noted that the optimum construction also becomes an angle ply since the optimum design point is located on the lower boundary curve. The optimum angles as functions of the plate aspect ratio are shown in Fig. 4. It is found that the optimum angles are very different between concentrated and distributed loading conditions.

The lamination parameter method is also capable of giving optimum design for the strength of laminates. Any strength constraint can be drawn on the ILP plane when the applied stresses are specified since the in-plane strains are the function of in-plane stiffness. In addition to the problems mentioned earlier, several applications of the lamination parameters are developed in tailoring laminated composites such as maximizing the natural frequency of laminates and minimizing the cost of laminated plates with sandwich construction under various constraints.

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